Entrance Syllabus for PhD admissions in Mathematics at University of Kashmir, Srinagar (Session: 2026-2027)

UNIT-I

Groups, Cyclic groups and the structure theorem for cyclic groups; Permutation Groups, General linear groups and Dihedral groups, Automorphism, inner and outer automorphisms, Cauchy's theorem and Sylow's theorems for abelian groups, symmetric groups, alternating groups and simplicity of the alternating group A_n, for $n \geq 5$; Conjugacy classes, the class equation of finite groups with applications; Cauchy's and Sylow's theorems for finite groups, Double cosets; direct product of groups; finite abelian groups; normal and subnormal series; composition series; Jordan–Hölder theorem; Zassenhaus lemma; Schreier's refinement theorem; solvable groups. Rings, integral domains and ideals; maximal, prime and principal ideals, example and relations, Field of quotients of an integral domain and embedding of an integral domain into its field of quotients.

UNIT-II

Euclidean rings with examples ; Principal ideal rings (PIR), unique factorization domains (UFD) and Euclidean domains; relationships among Euclidean rings, PIRs and UFDs; greatest common divisor and least common multiple in rings; polynomial rings and the division algorithm for polynomials; irreducible polynomials; primitive polynomials; Eisenstein's irreducibility criterion. Fields: Prime fields and their structure, extensions of fields, algebraic numbers and algebraic extensions of a field, roots of polynomials, remainder and factor theorems, splitting field of a polynomial, existence and uniqueness of splitting fields of polynomials , simple extension of a field.

UNIT-III

Riemann–Stieltjes integral: definition, existence, properties, and criteria for integrability; relation to the Riemann integral; integration by parts; connections with functions of bounded variation; fundamental theorem of calculus. Improper integrals: comparison tests, Cauchy's test, integrals over infinite intervals, absolute and conditional convergence, Abel's and Dirichlet's tests, integral tests for series, and convergence of oscillatory integrals. Inequalities: arithmetic–geometric mean inequality, weighted AM–GM inequality, Cauchy–Schwarz, Jensen, Hölder, and Minkowski inequalities; convexity and Jensen's inequality; Young's inequality; Chebyshev's inequality for monotone functions.

UNIT-IV

Complex Analysis: Continuity and differentiability of complex functions; Cauchy–Riemann equations and analytic functions; necessary and sufficient conditions for analyticity; harmonic functions and conjugate harmonic functions; Laplace's equation and construction of harmonic conjugates; complex integration and the Cauchy–Goursat theorem; Cauchy's integral formula and higher-order derivatives; Morera's theorem; Cauchy's inequality and Liouville's theorem with its generalizations; fundamental theorem of algebra; Taylor's theorem for analytic functions; Laurent series and classification of singularities; maximum modulus theorem; Schwarz lemma and its generalizations; zeros of analytic functions and their isolated nature; identity theorem and analytic continuation; argument principle and Rouche's theorem with applications; maximum and minimum modulus principles; Schwarz reflection principle.

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UNIT-V

Normed Linear Spaces and Banach Spaces: definitions and examples; subspaces and quotient spaces; continuous linear operators; completeness of L(X,Y); equivalent norms in finite-dimensional spaces; Hahn–Banach theorem and basic applications; dual spaces and examples; bounded linear functionals; uniform boundedness principle; open mapping theorem; closed graph theorem; Banach space isomorphisms; examples including C[a, b], lp and Lp spaces. Inner Product Spaces and Hilbert Spaces: definitions and examples; Cauchy–Schwarz inequality; orthogonality and orthonormal systems; Gram–Schmidt orthonormalization process; Bessel's inequality and Parseval's identity; orthonormal bases and completeness in separable Hilbert spaces; projection theorem and best approximation; Riesz representation theorem; adjoint of an operator; self-adjoint, unitary, and normal operators; spectral theorem for compact self-adjoint operators.

UNIT-VI

First-order ODEs, singular solutions, p-discriminant and c-discriminant, and initial value problems; theory of homogeneous and non-homogeneous linear ODEs; simultaneous linear equations with constant coefficients, normal form and factorization of operators; method of variation of parameters; Picard's existence and uniqueness theorem. Series solutions of ODEs: indicial equations with unequal roots, equal roots, roots differing by a non-integer, and roots differing by an integer leading to infinite or indeterminate coefficients. Simultaneous equations and their solutions using multipliers; finding a second integral from the first; Differential equations of the type Pdx+Qdy+Rdz=0; necessary and sufficient conditions for integrability and geometric interpretation of such equations.

UNIT-VII

Partial Differential Equations (PDEs): first-order linear and nonlinear PDEs; Lagrange's method for linear equations; Charpit's and Jacobi methods for nonlinear equations; initial-value problems for quasi-linear first-order PDEs and Cauchy's method of characteristics; characteristic curves. Origin and classification of second-order PDEs; linear second-order PDEs with constant coefficients; Monge's method; methods of solution and canonical forms. Classical equations: Laplace, Heat, and Wave equations; solutions using separation of variables, Fourier series, and integral transform methods; Laplace transform methods; D'Alembert's method; Duhamel's principle. Boundary value problems and initial-boundary value problems; superposition principle; applications to physical models in mechanics and mathematical physics.

UNIT-VIII

Countable and uncountable sets, Schroeder–Bernstein theorem, the axiom of choice and equivalent forms; metric spaces with examples, open and closed sets, completeness and Baire's Category Theorem, with applications including the non-existence of a function that is continuous exactly on the irrationals, and the impossibility of approximating the characteristic function of the rationals on [0,1] by continuous functions. Completion of metric spaces, Cantor's intersection theorem with examples illustrating the necessity of its conditions, uniformly continuous mappings with examples and counterexamples, extension of uniformly continuous maps, and Banach's contraction principle with applications such as the inverse function theorem in R.

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UNIT-IX

Differentiable and regular curves; parameterizations; arc-length and unit-speed curves. Plane curves: curvature, osculating circle, centre of curvature, directed curvature, and examples including the line, circle, ellipse, and tractrix. Evolutes and involutes. Space curves: Frenet–Serret frame, curvature, torsion, intrinsic equations, and the characterization of helices and spherical curves. Surfaces: regular surfaces and coordinate charts; change of coordinates; tangent plane and normal vector; orientability; differentiable mappings and differentials; first fundamental form, line element and its invariance; angle between curves; orthogonality of coordinate curves; and area of a region with invariance under coordinate transformations.

UNIT-X

Characteristic and minimal polynomials, Cayley–Hamilton theorem and applications. Conditions for diagonalizability of an n × n matrix, Orthogonal reduction of real matrices, Orthogonality of eigenvectors of Hermitian matrices, Unitary and orthogonal diagonalization, Quadratic forms, positive definiteness, Rank, index, and signature of a quadratic form.

Euler graphs and Euler's theorem, Hamiltonian graphs and Dirac's theorem, degree sequences. Trees and their properties, centres in trees, binary and spanning trees, Cayley's theorem, fundamental cycles, signed graphs, Cut-sets and their properties, vertex connectivity, edge connectivity, Whitney's theorem, Menger's theorem (vertex and edge form), planar graphs, Kuratowski's two graphs, Euler's formula, Kuratowski's theorem, geometric dual, regular polyhedras. Incidence matrix A(G), cycle matrix B(G), cut-set matrix C(G).

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