

Sr. No.100.....

ENTRANCE TEST-2023

SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCES

OPTION—1 : STATISTICS

OPTION—2 : MATHEMATICS FOR STATISTICS

Total Questions : 60

Question Booklet Series

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Time Allowed : 70 Minutes

Roll No. :

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Instructions for Candidates :

1. Write your Entrance Test Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be '**Negative Marking**' for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer Sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

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OPTION—1 : STATISTICS

1. At every iteration of simplex method, for a minimization problem, a variable in current basis is replaced with another variable that has :
 - (A) A negative value of $Z_j - C_j$
 - (B) The value of $Z_j - C_j = 0$
 - (C) A positive value of $Z_j - C_j$
 - (D) None
2. A set of values of decision variables which satisfies the linear constraints and non-negativity conditions of a LPP is known as :
 - (A) Solution
 - (B) Feasible solution
 - (C) Optimal solution
 - (D) Unbounded solution
3. The optimal value of the objective function is attained at the points :
 - (A) Given by intersection of lines representing inequations with axes only
 - (B) Given by intersection of lines representing inequations with X-axis only
 - (C) Given by corner points of the feasible region
 - (D) At the origin
4. If the two constraints do not intersect in the positive quadrant of the graph, then :
 - (A) The problem is infeasible
 - (B) The solution is unbounded
 - (C) One of the constraints is redundant
 - (D) None of these
5. In fitting a straight line, the value of slope b remains unchanged with the change of :
 - (A) Scale
 - (B) Origin
 - (C) Both (A) and (B)
 - (D) Neither (A) Nor (B)
6. Secular trend is indicative of long-term variation towards :
 - (A) Increase only
 - (B) Either increase or decrease
 - (C) Decrease only
 - (D) None of these
7. The trend values in freehand curve method are obtained by :
 - (A) Equation of straight line
 - (B) Second degree parabola
 - (C) Graph
 - (D) All of these
8. For the given data semi averages for the second half is given by :

Year	2010	2011	2012	2013	2014	2015	2016	2017
Sales	20	16	9	11	40	23	21	12

 - (A) 14
 - (B) 22
 - (C) 23
 - (D) 24
9. The rate computed by adding the age specific fertility rates of various age groups of child bearing age is known as :
 - (A) Crude Birth Rate
 - (B) Net Reproduction Rate
 - (C) Total Fertility Rate
 - (D) General Fertility Rate
10. In a given year, the Crude Birth Rate of population of size 1,50,000 is 12. The number of births is :
 - (A) 18
 - (B) 180
 - (C) 1800
 - (D) 18000

11. The population will increase, remains stationary or decreasing according to whether the NRR exceeds, equal or is less than :
- 0
 - 0.5
 - 1
 - 1.5
12. The data related to births, deaths, marriage and divorce is called :
- Mortality
 - Morbidity
 - Vital statistics
 - Survey
13. If the sample values lie within the control limits and are in a random way, we say that the process is :
- Under control
 - Under warning limits
 - Out of control
 - Need to stop the process
14. If a characteristic follows normal distribution, then 3 sigma limits covers _____ of observations.
- 0.27%
 - 68.26%
 - 95.44%
 - 99.73%
15. The chart used to monitor attributes is :
- Range chart
 - C-chart
 - Mean chart
 - All of the above
16. Which of the following is a type of control chart used in Statistical Quality Control ?
- Histogram
 - Box and Whisker Plot
 - Scatter Plot
 - \bar{X} chart
17. What is estimation in Statistics ?
- The process of making a prediction or approximation about a population parameter based on a sample of data
 - The process of accurately measuring a population parameter
 - The process of collecting data on a population
 - The process of analysing data to draw conclusions
18. A confidence interval will be widened if :
- The confidence level is decreased and the sample size is increased
 - The confidence level is increased and the sample size is reduced
 - The confidence level is increased and the sample size is increased
 - The confidence level is decreased and the sample size is decreased
19. Sampling distribution is :
- A distribution of the population parameters
 - A distribution of the sample statistics
 - A distribution of the individual data points in sample
 - A distribution of the differences between the sample and population parameters
20. A function for estimating a parameter is called as :
- Estimate
 - Estimation
 - Estimator
 - None of these
21. The coefficient of variation is 58%. If mean is 10 then standard deviation is :
- 5.8
 - 580
 - 0.17
 - None of these

22. If the distribution is negatively skewed, then :
- (A) Mean is more than the mode
 - (B) Mean is less than the mode
 - (C) Median is at right to the mode
 - (D) Mean is at right to the median
23. The Median of scores 25, 45, 35, 35, 40, 30 is :
- (A) 45
 - (B) 40
 - (C) 35
 - (D) 30
24. The arithmetic mean of the first ten whole numbers is :
- (A) 5.5
 - (B) 5
 - (C) 4
 - (D) 4.5
25. The correlation coefficient between two variables X and Y is 0.4. The correlation between 2X and (-Y) will be :
- (A) 0.4
 - (B) -0.8
 - (C) -0.4
 - (D) 0.8
26. In regression analysis, the variable that is used to explain the change in the outcome of an experiment is called :
- (A) The independent variable
 - (B) The predictor variable
 - (C) The explanatory variable
 - (D) All of the above
27. If the coefficient of determination is equal to 1. Then correlation coefficient :
- (A) Must also be equal to 1
 - (B) Can either be -1 or +1
 - (C) Can be any value between -1 to 1
 - (D) Must be -1
28. If $r = 0.8$, $b_{yx} = 1.5$, then b_{xy} is approximately :
- (A) 0.32
 - (B) 0.42
 - (C) 0.75
 - (D) 1
29. Suppose $P(X) = 0.36$ and $P(Y) = 0.41$. If $P(X|Y) = 0.27$, what is $P(Y|X)$?
- (A) $\frac{(0.36)(0.41)}{(0.27)}$
 - (B) $\frac{(0.27)(0.41)}{(0.36)}$
 - (C) $\frac{(0.36)(0.27)}{(0.41)}$
 - (D) $\frac{(0.27)}{0.36 + 0.41}$
30. If $P(A) = 0.32$ and $P(B) = 0.45$, what is $P(A \cup B)$ if A and B are independent ?
- (A) 0.144
 - (B) 0.626
 - (C) 0.770
 - (D) 0.856
31. The probability of throwing 10 with 2 dice is :
- (A) 1/6
 - (B) 1/12
 - (C) 2/3
 - (D) 1/4
32. Which of the following can be the probability of an event ?
- (A) -1.3
 - (B) 0.04
 - (C) 3/8
 - (D) 10/7

33. Let X have pmf

$$f(x) = \frac{x}{10}, x = 1, 2, 3, 4$$

Then E(X) is equal to :

- (A) 3
- (B) 6
- (C) 9
- (D) 12

34. Given $E(X + 4) = 10$ and $E[(X + 4)^2] = 116$. Then $V(X + 4)$ is :

- (A) 4
- (B) 16
- (C) 3
- (D) 9

35. The variance of probability distribution

X	-3	-2	-1	0	1	2	3
P(X)	1/7	1/7	1/7	1/7	1/7	1/7	1/7

is :

- (A) 0
- (B) 8
- (C) 4
- (D) 12

36. The Mean of the distribution

$$\binom{10}{x} \left(\frac{2}{5}\right)^x \left(\frac{3}{5}\right)^{10-x}, x = 0, 1, 2, \dots, 10$$

is :

- (A) 4
- (B) 5
- (C) 6
- (D) 10

37. Let X have the Uniform pdf $U(0, 100)$, then variance of X is given by :

- (A) 1/12
- (B) 100/12
- (C) 1000/12
- (D) 10000/12

38. The point of inflexion of normal curve are :

- (A) $\mu \pm \sigma$
- (B) $\mu \pm 3\sigma$
- (C) $\mu \pm 2\sigma$
- (D) None of these

39. Match the following :

Distribution	MGF
a. Normal	1. $e^{\lambda(e^t - 1)}$
b. Gamma	2. $e^{\mu t + t^2 \sigma^2 / 2}$
c. Poisson	3. $(1 - t)^{-\lambda}$

- (A) a-2, b-1, c-4
- (B) a-3, b-1, c-2
- (C) a-1, b-3, c-2
- (D) a-2, b-3, c-1

40. If $X_1, X_2, X_3, \dots, X_n$ are independent and have normal distributions $N(\mu_i, \sigma_i^2)$, $i = 1, 2, 3, \dots, n$, respectively. Then the distribution of

$$W = \sum_{i=1}^n \left(\frac{X_i - \mu_i}{\sigma_i} \right)^2 \text{ is :}$$

- (A) Gamma distribution
- (B) Chi-square distribution
- (C) Normal distribution
- (D) t-distribution

41. In a test with a standard deviation of 12 and mean 44 a student scored 41 marks. His Z score is :

- (A) 0.50
- (B) -0.50
- (C) 0.25
- (D) -0.25

42. Arrange the following steps in process of hypothesis testing in proper sequence :
- Select the level of significance
 - Setup null and alternative hypothesis
 - Establish the decision rule
 - Perform computation
 - Select test statistics
 - Draw decision
- (A) a, b, c, d, e, f
 (B) a, b, e, d, c, f
 (C) b, a, c, d, e, f
 (D) b, a, e, c, d, f
43. The dividing point between the region where the null hypothesis is rejected and the region where it is not rejected is said to be :
- (A) Critical region
 (B) Significance value
 (C) Critical value
 (D) Acceptance region
44. Which of the following statements best describes on type I error ?
- (A) Rejecting a null hypothesis when it is true
 (B) Failing to reject a false null hypothesis
 (C) Accepting a true alternative hypothesis
 (D) Rejecting a false alternative hypothesis
45. For testing of goodness of fit :
- (A) The expected frequency should exceed 5
 (B) The observed frequency should exceed 5
 (C) Both the expected and observed frequency should exceed 5
 (D) None of the above conditions are necessary
46. What type of data do you need for a Chi square test ?
- (A) Ordinal
 (B) Interval
 (C) Ratio
 (D) Categorical
47. A Medical Assistant sampled the blood pressure of 20 randomly selected patients with high blood pressure and after receive a dose of a new medicine, which hypothesis test should she run ?
- (A) t-test for single mean
 (B) F-test for equality of variances
 (C) Independent t-test for difference of means
 (D) Paired t-test
48. Which of the following are conditions for using the t-distribution for small sample difference tests ?
- Samples must be independent.
 - Samples must be drawn from normal populations.
 - Samples must be of equal size.
- (A) I only
 (B) I and II
 (C) II only
 (D) I and III
49. The complete list of population, where each sampling unit is identified by a number is known as :
- (A) Voter list
 (B) Sampling frame
 (C) A list of random numbers
 (D) None of these
50. The relative efficiency of SRSWOR with SRSW is :
- (A) $\frac{N-n}{N-1}$
 (B) $\frac{N-n}{N}$
 (C) $\frac{n}{N}$
 (D) $\frac{N-1}{N}$

51. In simple random sampling, the sample mean is :
- Always zero
 - Smaller than population mean
 - Equal to population mean
 - Random variable
52. The finite population correction in usual notation is expressed as :
- $(N - n)/N$
 - $1 - (n/N)$
 - Both (A) and (B)
 - None
53. Which of the following is not true ?
- $\text{Var}(\bar{y}_{st})_P \geq \text{Var}(\bar{y}_{st})_{Ney}$
 - $\text{Var}(\bar{y})_{SRS} \geq \text{Var}(\bar{y}_{st})_P$
 - $\text{Var}(\bar{y})_{SRS} \leq \text{Var}(\bar{y}_{st})_{Ney}$
 - All of these
54. In proportional allocation we have :
- $n_i = \frac{n}{N} N_i$
 - $n_i = n/k$
 - $n_i = N/k$
 - $n_i = \frac{n}{N_i} N$
55. Which of the following is an example of systematic sampling ?
- A researcher selects every 10th person who enters a shopping mall to participate in a study
 - A researcher selects a random sample of participants from a list of registered voters
 - A researcher selects a convenience sample of participants from a local community center
 - None of these
56. The purpose of stratified sampling is to :
- To ensure that the sample is representative of the population
 - To save time and money by not having to sample the entire population
 - To obtain a biased sample
 - To obtain non-random sample
57. For analysing the completely randomised design with t treatments each replicated r times each, with one missing observation, total degree of freedom are :
- Rt
 - $rt - 1$
 - $rt - 2$
 - $(r - 1)(t - 1)$
58. If the total degrees of freedom and between treatments degrees of freedom in a completely randomized design are 15 and 4 respectively, the degrees of freedom for error will be :
- 11
 - 14
 - 18
 - 19
59. How many factors are involved in a randomized block design ?
- One
 - Two
 - Three
 - Four
60. In ANOVA, what is the null hypothesis ?
- There is no difference between the means of the groups
 - There is a difference between the means of the groups
 - The groups are not normally distributed
 - The sample size is too small to draw a conclusion

OR

OPTION—2 : MATHEMATICS FOR STATISTICS

1. Which of the following is not true ?

(A) A positive term series either converges or diverges to $+\infty$

(B) The series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if $\lim_{n \rightarrow \infty} a_n = 0$

(C) The series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the sequence of its partial sums is convergent

(D) The sum of two convergent series is convergent

2. For what value of x is the series

$$1 + x + x^2 + x^3 + \dots$$

divergent ?

(A) $x < -1$

(B) $x = -1$

(C) $x \geq 1$

(D) $-1 < x < 1$

3. For what value of x is the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$ convergent ?

(A) $x < 1$

(B) $x > 1$

(C) $x = 1$

(D) None of these

4. Which of the following is true for the series

$$\sum_{n=1}^{\infty} a_n \text{ and } \sum_{n=1}^{\infty} b_n, \text{ where } a_n = \frac{(-1)^{n-1}}{n} \text{ and}$$

$$b_n = \frac{(-1)^{n-1}n}{2n-1} ?$$

(A) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent

(B) Neither $\sum_{n=1}^{\infty} a_n$ Nor $\sum_{n=1}^{\infty} b_n$ is convergent

(C) $\sum_{n=1}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} b_n$ is divergent

(D) $\sum_{n=1}^{\infty} a_n$ is divergent but $\sum_{n=1}^{\infty} b_n$ is convergent

5. Which of the following series is/are uniformly convergent for all real values of x and $0 < r < 1$?

(A) $\sum_{n=1}^{\infty} r^n \cos nx$

(B) $\sum_{n=1}^{\infty} r^n \sin nx$

(C) $\sum_{n=1}^{\infty} r^n \cos n^2x$

(D) All of the above

6. Which of the following is true for the sequence $\{f_n\}$, where $f_n(x) = \tan^{-1}(nx)$, $x \geq 0$, $n = 1, 2, 3, \dots$?

(A) $\{f_n\}$ is not uniformly convergent in any interval $[a, b]$, $a > 0$

(B) $\{f_n\}$ is uniformly convergent in $[0, b]$

(C) $\{f_n\}$ is not point-wise convergent in $[0, b]$

(D) None of these

7. Which of the following is not true for the sequence $\{f_n\}$, where $f_n(x) = nxe^{-nx^2}$, $n = 1, 2, 3, \dots$ and $f(x) = \lim_{n \rightarrow \infty} f_n(x)$?
- (A) $\int_0^1 f(x) dx = 0$
- (B) $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \frac{1}{2}$
- (C) $f(x) = 0, \forall x$
- (D) $\{f_n\}$ does not converge uniformly to f
8. The radius of convergence of the power series $1 + 2x + 3x^2 + 4x^3 + \dots$ is :
- (A) 1
- (B) ∞
- (C) 0
- (D) 2
9. Which of the following is a group?
- (A) The set of natural numbers under addition
- (B) The set of real numbers under addition
- (C) The set of real numbers under multiplication
- (D) The set of complex numbers under multiplication
10. Which of the following conditions makes any multiplicative group G into an Abelian group?
- (A) $\forall a, b \in G, (a \cdot b)^2 = a^2 \cdot b^2$
- (B) Each element of G is idempotent
- (C) Each element of G is its own inverse
- (D) All of the above
11. What is the order of the symmetric group S_4 ?
- (A) 4
- (B) 6
- (C) 24
- (D) 120
12. The number of elements in the quaternion group is equal to :
- (A) 4
- (B) 6
- (C) 8
- (D) 24
13. Which of the following is not true?
- (A) A non-empty subset S of a group G is a subgroup of G if and only if $\forall a, b \in S, ab^{-1} \in S$
- (B) For any subgroup S of a group $G, O(S) | O(G)$
- (C) For any element a in a group $G, a^{O(G)} = e$
- (D) For any two subgroups A and B of $G, A \cup B$ is also a subgroup of G
14. Which of the following is a generator of the cyclic group formed by the n^{th} roots of unity?
- (A) $e^{\frac{2\pi i}{n}}$
- (B) $e^{\frac{\pi i}{n}}$
- (C) $e^{2n\pi}$
- (D) None of these
15. Which of the following is not true for the multiplicative group $G = \{1, -1, i, -i\}$, where $i = \sqrt{-1}$?
- (A) G is an abelian group
- (B) G is a cyclic group with two generators
- (C) $O(i) = 4$
- (D) G has no subgroup of order 2

16. The order of the quotient group $\frac{Z}{N}$, where Z is the additive group of integers and $N = \langle 3 \rangle$, the subgroup of Z consisting of all multiples of 3, is :

- (A) 1
- (B) 2
- (C) 3
- (D) Infinity

17. Which of the following rings is not free from zero divisors ?

- (A) Ring of integers
- (B) Ring of all square matrices of order 2
- (C) Ring of integers modulo a prime number
- (D) Ring of complex numbers

18. If f and g are any two non-zero polynomials over a ring R without proper zero divisors, then $\text{degree}(f \cdot g)$:

- (A) is less than $\max(\text{deg } f, \text{deg } g)$
- (B) is less than $\text{deg } f + \text{deg } g$
- (C) is equal to $\text{deg } f + \text{deg } g$
- (D) is equal to $\text{deg } f \cdot \text{deg } g$

19. Which of the following is not true ?

- (A) Every finite integral domain is a field
- (B) Every field is an integral domain
- (C) Every ideal of a ring R is a subring of R
- (D) The ring of integers is an ideal of the ring of rational numbers

20. Which of the following is true for any element a in a ring R and the subset $S = \{x \in R; xa = 0\}$?

- (A) S is a subring of R
- (B) S is a right ideal of R
- (C) S is an ideal of R
- (D) None of these

21. If

$$f(x) = \frac{x}{|x|}, x \neq 0$$

$$= 0 \text{ if } x = 0,$$

then $\lim_{x \rightarrow 0} f(x) =$

- (A) 0
- (B) 1
- (C) -1
- (D) None of these

22. What type of discontinuity does the function

$$f(x) = e^{-\frac{1}{x}}$$

have at $x = 0$?

- (A) Removable discontinuity
- (B) Discontinuity of the first kind
- (C) Discontinuity of the second kind
- (D) None of these

23. Which of the following is true for the function

$$y = \log(x + \sqrt{1+x^2}) ?$$

- (A) $xy_2 + y_1 = 0$
- (B) $(1+x^2)y_2 + y_1 = 0$
- (C) $(1+x^2)y_2 + xy_1 = 1$
- (D) $(1+x^2)y_2 + xy_1 = 0$

24. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$

- (A) $\cos u$
- (B) $2 \cot u$
- (C) $\frac{1}{2} \cot u$
- (D) $-\frac{1}{2} \cot u$

25. The angle of intersection of the curves $r = a\theta$ and $r = \frac{a}{\theta}$ is :

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{2}$

26. The angle between the radius vector and the tangent to the curve $r = a(1 - \cos \theta)$ at any point is equal to :

(A) θ

(B) 2θ

(C) $\frac{\theta}{2}$

(D) $\frac{\theta}{3}$

27. The radius of curvature of the curve $x^2 + y^2 = 2$ at the point (1, 1) is :

(A) $\sqrt{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) 2

(D) $\frac{1}{2}$

28. The number of asymptotes to the curve

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

is :

(A) 2

(B) 1

(C) 4

(D) None

29. Which of the following is/are true for the function

$$f(x) = \sin x, x \in I = \left[0, \frac{\pi}{2}\right] ?$$

(A) There is a value of $x \in I$ where $f(x) = \frac{1}{3}$

(B) The maximum value of $f(x)$ in I is 1

(C) There is a value of $x \in I$ where $f'(x) = 0$

(D) All of the above

30. The coefficient of x^n in the Maclaurin's series of $-\log(1 - x)$ is :

(A) $\frac{1}{n}$

(B) $-\frac{1}{n}$

(C) $\frac{1}{n!}$

(D) $-\frac{1}{n!}$

31. If $f(x) = x^2 - 3x - 1$, $x \in [1, 3]$, then the value of the "c" of the Mean Value Theorem is :

- (A) 2.75
- (B) 2.5
- (C) 2.3
- (D) 2.1

32. The value of $\lim_{x \rightarrow +\infty} (1+x)^{\frac{1}{x}}$ is :

- (A) 0
- (B) 1
- (C) e
- (D) $\frac{1}{e}$

33. The value of $\int_0^1 \frac{1}{2e^x - 1} dx$ is :

- (A) $\log 2$
- (B) $\log\left(\frac{2e-1}{e}\right)$
- (C) 1
- (D) 0

34. The integral of $\sqrt{x} \tan^{-1} \sqrt{x}$ with respect to x is equal to :

- (A) $x^{\frac{3}{2}} \tan^{-1} \sqrt{x} - \frac{x}{2} + \frac{1}{2} \log(1+x) + c$
- (B) $\frac{3}{2} \left[x^{\frac{3}{2}} \tan^{-1} \sqrt{x} - \frac{x}{2} + \frac{1}{2} \log(1+x) \right] + c$
- (C) $\frac{2}{3} \left[x^{\frac{3}{2}} \tan^{-1} \sqrt{x} - \frac{x}{2} + \frac{1}{2} \log(1+x) \right] + c$
- (D) $2 \left[x^{\frac{3}{2}} \tan^{-1} \sqrt{x} - \frac{x}{2} + \frac{1}{2} \log(1+x) \right] + c$

35. Which of the following is/are true for

$$I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx ?$$

- (A) $n(I_{n-1} + I_{n-2}) = 1$
- (B) $I_n + I_{n-2} = \frac{1}{n-1}$
- (C) $I_1 = \log \sqrt{2}$
- (D) All of the above

36. What is the value of $\int_0^{\infty} \frac{dx}{(1+x^2)^2}$?

- (A) $\frac{\pi}{4}$
- (B) $\frac{\pi}{2}$
- (C) $\frac{3\pi}{2}$
- (D) $\frac{3\pi}{4}$

37. Which of the following substitutions reduces the differential equation $xy(1+xy^2) \frac{dy}{dx} = 1$ to a linear differential equation ?

- (A) $\frac{1}{x} = z$
- (B) $\frac{1}{y} = z$
- (C) $xy = z$
- (D) $y = x^2$

38. Which of the following is a necessary and sufficient condition for the differential equation $M dx + N dy = 0$ to be exact ?
- (A) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$
- (B) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
- (C) Neither (A) Nor (B)
- (D) Both (A) and (B)
39. Which of the following is the particular integral of the differential equation $(D^3 - 3D^2 + 4)y = e^{2x}$?
- (A) $x^2 e^{2x}$
- (B) $\frac{x}{6} e^{2x}$
- (C) $\frac{x^2}{6} e^{2x}$
- (D) $\frac{x^2}{2} e^{2x}$
40. The complementary function of the differential equation
- $$(3x + 2)^2 \frac{d^2 y}{dx^2} + 3(3x + 2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$$
- is :
- (A) $c_1(3x + 2)^2 + \frac{c_2}{(3x + 2)^2}$
- (B) $c_1(3x + 2) + \frac{c_2}{(3x + 2)}$
- (C) $c_1(3x + 2)^2 + c_2(3x + 2)$
- (D) $c_1(3x + 2) + \frac{c_2}{(3x + 2)^2}$
41. The general solution of the differential equation $p^2 + 2py \cot x = y^2$ is given by :
- (A) $\frac{y}{1 \pm \cos x} = c$
- (B) $y(1 \pm \cos x) = c$
- (C) $x(1 \pm \cos y) = c$
- (D) $y(1 \pm \sin x) = c$
42. The general solution of the differential equation $p = \tan\left(x - \frac{p}{1 + p^2}\right)$ is given by :
- (A) $x = \tan^{-1} p + \frac{p}{1 + p^2}, y = c - \frac{1}{1 + p^2}$
- (B) $x = c - \frac{1}{1 + p^2}, y = \tan^{-1} p + \frac{p}{1 + p^2}$
- (C) Neither (A) Nor (B)
- (D) Both (A) and (B)
43. The solution of the differential equation $yp + px - p^2x - y = p$ is given by :
- (A) $y = x + c$
- (B) $y = \frac{x^2}{2} + c$
- (C) $y = cx + \frac{c}{c - 1}$
- (D) $y = cx$
44. The substitution $X = x^2, Y = y^2$ reduces the equation $(px - y)(py + x) = 2p$ to :
- (A) Linear form
- (B) Bernoulli's form
- (C) Clairut's form
- (D) None of these

45. The values of the Bessel's functions $J_0(x)$ and $J_1(x)$ at $x = 0$ are respectively :
- (A) 0 and 1
 (B) 1 and 0
 (C) 0 and 0
 (D) 1 and 1
46. Which of the following is true ?
- (A) $J_2(x) = J_1(x) - J_0(x)$
 (B) $J_2(x) = J_1(x) + J_0(x)$
 (C) $J_2(x) = \frac{x}{2}J_1(x) - J_0(x)$
 (D) $J_2(x) = \frac{2}{x}J_1(x) - J_0(x)$
47. Which of the following is not true for the Legendre polynomial $P_n(x)$?
- (A) $P_n(1) = 1$
 (B) $P_n(-x) = (-1)^n P_n(x)$
 (C) $P_{2n+1}(0) = 0$
 (D) $P_{2n}(0) = \frac{(-1)^n (2n)!}{(n!)^2}$
48. For $m = n$, $\int_0^1 P_m(x)P_n(x) dx =$
- (A) 0
 (B) 1
 (C) $\frac{2}{2n+1}$
 (D) $\frac{2}{n+1}$
49. The order and the degree of the partial differential equation $\frac{\partial^2 u}{\partial x \partial y} = \left(\frac{\partial u}{\partial z}\right)^3$ are respectively :
- (A) 2 and 2
 (B) 2 and 1
 (C) 1 and 2
 (D) 2 and 3
50. The solution of the partial differential equation $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial y} = 1$, is :
- (A) $z = e^x \cos y + \sin y$
 (B) $z = e^y \cos y + \sin x$
 (C) $z = e^y \cos x + \sin x$
 (D) $z = e^x \sin y + \cos x$
51. The general solution of the partial differential equation $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ is :
- (A) $x^2 + y^2 + z^2 = f(x+y+z)$
 (B) $xyz = f(x^2 + y^2 + z^2)$
 (C) $f\left(x^2 + y^2 + z^2, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$
 (D) $f\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, xyz\right) = 0$
52. The complete solution of partial differential equation $p = e^q$ is :
- (A) $z = ax + y \log a + c$
 (B) $z = ax + \log y + c$
 (C) $z = ax + by + c$
 (D) $z = ay + x \log a + c$

53. Which of the following sets is not countable ?
- (A) The set of natural numbers
 (B) The set of rational numbers
 (C) The set of complex numbers with rational real and imaginary parts
 (D) The set of points in the closed interval $[1, 2]$
54. Which of the following is a bounded set ?
- (A) The set of natural numbers
 (B) The set of rational numbers
 (C) The set $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$
 (D) The set of positive real numbers
55. Which of the following is true ?
- (A) Every infinite set has a limit point
 (B) Every bounded set has a limit point
 (C) Every finite set has a limit point
 (D) Every infinite bounded set has a limit point
56. If for any real numbers x, y and a , $|x - a| < \epsilon$, $|y - a| < \epsilon$, then $|x - y| <$
- (A) ϵ
 (B) $\frac{\epsilon}{2}$
 (C) 2ϵ
 (D) $a + \epsilon$
57. For what value of a_n the sequence $\{a_n\}$ oscillates infinitely ?
- (A) $a_n = (-1)^n$
 (B) $a_n = (-1)^{n^2}$
 (C) $a_n = n^2$
 (D) $a_n = -2^n$
58. The limit of the sequence $\{a_n\}$, where
- $$a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \dots + \frac{1}{3^n},$$
- is :
- (A) 1
 (B) $\frac{1}{3}$
 (C) $\frac{2}{3}$
 (D) $\frac{3}{2}$
59. For what of a_n is the sequence $\{a_n\}$ a monotonic decreasing sequence that is not convergent ?
- (A) $a_n = \frac{1}{n}$
 (B) $a_n = -n$
 (C) $a_n = \frac{n+1}{n}$
 (D) $a_n = \frac{n}{n^2 + 1}$
60. If b and c are positive real numbers, then the sequence $\left\{\left(1 + \frac{b}{n}\right)^{cn}\right\}$ converges to :
- (A) e
 (B) e^b
 (C) e^c
 (D) e^{bc}