Sr. No. 379

ENTRANCE TEST-2023

SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCE

MATHEMATICS

Total Questions : 60
Time Allowed : 70 Minutes

Question Booklet Series C

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Entrance Test Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be **'Negative Marking'** for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer Sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/ herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

SEAL

1. What is the radius of curvature at any point of the curve $r = ae^{\theta \cot\alpha}$?

(A) $r \operatorname{cosec}\alpha$

(B) $r \cot\alpha$

(C) $r \sec\alpha$

(D) None of the above

2. The asymptotes of the curve $r\theta = a$ are :

(A) $r \sin\theta = a$

(B) $r \cos\theta = a$

(C) $\cos\theta = r^2$

(D) None of the above

3. If $U = \sqrt{x^2 - y^2}\sin^{-1}\frac{y}{x}$, then $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y}$ equals :

(A) 2U

(B) 0

(C) U

(D) None of the above

4. If $Z = x^y$, then $\frac{\partial Z}{\partial x}$ equals :

(A) $x^y \log x$

(B) $y\, x^{y-1}$

(C) $y\, e^x$

(D) None of the above

5. What is the nth-derivative of 3^{2x} ?

(A) $y_n = 3^{2x}(\log 3)^n$

(B) $y_n = 2(3^{2x})^n(\log 3)$

(C) $y_n = 2^n 3^{2x}(\log 3)^n$

(D) None of the above

6. $\frac{d}{dx}(\sin\sqrt{x})$ equals :

(A) $\cos\sqrt{x}$

(B) $\frac{1}{2\sqrt{x}}\cos\sqrt{x}$

(C) $\frac{\sqrt{x}}{2}\cos\sqrt{x}$

(D) None of the above

7. Let the functions f(x) and g(x) be defined by :

$$f(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$

$$g(x) = \begin{cases} 0, & \text{if } x \geq 0 \\ 1, & \text{if } x < 0 \end{cases}$$

Then

(A) f(x) is continuous at x = 0

(B) g(x) is continuous at x = 0

(C) f(x) + g(x) is continuous at x = 0

(D) f(x)g(x) is discontinuous at x = 0

8. If $x = a\cos\theta$ and $y = b\sin\theta$, then the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ is :

(A) $-\frac{b}{a^2}$

(B) ab

(C) $-\frac{b}{a}$

(D) None of the above

9. What is the value of the function $f(x) = x^2e^x$?

(A) $4e^{-2}$

(B) 0

(C) -4

(D) None of the above

10. Rolle's theorem cannot be applied to the function $f(x) = |x|$ in $[-1, 1]$ because :

(A) the function is discontinuous at $x = 0$

(B) the function is not derivable at $x = 0$

(C) the function is discontinuous at $x = \frac{1}{2}$

(D) the function is not derivable at $x = -\frac{1}{2}$

11. The coefficient of x in the Maclaurin's series of $f(x) = a^x$ is :

(A) 1

(B) log a

(C) a

(D) $(\log a)^2$

12. $\int \frac{1}{x - x^3} dx$ equals :

(A) $\log\sqrt{1-x^2}$

(B) $\log x - \log\sqrt{1-x^2}$

(C) $\log x - \log(1-x^2)$

(D) $\log x + \log\sqrt{1-x^2}$

13. $\int_1^2 \log x \, dx$ equals :

(A) log 4 – 1

(B) 2 log 2

(C) 2 log 2 + 1

(D) None of the above

14. The reduction formula of $I_n = \int \tan^n x \, dx$ is

(A) $I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$

(B) $I_n = \frac{\tan^{n-2} x}{n-2} - I_{n-1}$

(C) $I_n = \frac{2\tan^n x}{n} - I_{n+3}$

(D) None of the above

15. For odd n, $\int_0^{\pi} \frac{\sin nx}{\sin x} dx$ equals :

(A) 0

(B) $-\pi$

(C) π

(D) None of the above

16. What is the general solution of the differential equation

$$\frac{dy}{dx} = e^{x-y} + x^2e^{-y} \ ?$$

(A) $y = \frac{x^3}{3} + e^x + c$

(B) $e^y = \frac{x^3}{3} + e^x + c$

(C) $y = e^x + e^{-y} + c$

(D) None of the above

17. What is the particular integral of the differential equation

$(D^2 - 3D + 2)y = e^x + e^{2x}$?

(A) $\frac{e^x}{5} + \frac{xe^{2x}}{6}$

(B) $-xe^x + \frac{xe^{2x}}{4} + c$

(C) $xe^{2x} - xe^x$

(D) None of the above

18. What is the complementary function of the differential equation

$(D^4 - D^2)y = 2$?

(A) $c_1e^x + c_2e^{-x}$

(B) $c_1 + c_2x + c_3e^x$

(C) $c_1 + c_2x + c_3e^x + c_4e^{-x}$

(D) None of the above

19. What is the general solution of the differential equation

$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$?

(A) $y = (c_1 + c_2 \log x)x^2$

(B) $y = (c_1e^x + c_2e^{-x})x$

(C) $y = (c_1 + c_2 \log x)$

(D) None of the above

20. What is the singular solution of the differential equation $y = px + \frac{a}{p}$, where $p = \frac{dy}{dx}$?

(A) $y^2 = 4ax$

(B) $y = 4ax^2$

(C) $y = 4ax$

(D) None of the above

21. What is the general solution of the differential equation $(px - y)^2 = p^2 + m^2$, where $p = \frac{dy}{dx}$.

(A) $y = cx \pm \sqrt{m^2 + c^2}$

(B) $y = \sqrt{cx - m}$

(C) $y = \sqrt{m^2 - 4ac}$

(D) None of the above

22. For a first order non-linear differential equation $f\left(x, y, \frac{dy}{dx}\right) = 0$, which of the following statements is/are true ?

(i) Its general solution must contain only one arbitrary constant.

(ii) Its singular solution can be obtained by substituting particular values of the arbitrary constant in its general solution.

(iii) Its singular solution is an envelope of its general solution which also satisfies the equation.

(A) (i), (ii) and (iii)

(B) (i) and (ii)

(C) (i) and (iii)

(D) (ii) and (iii)

23. Which of the following is a non-linear differential equation ?

(A) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = e^x$

(B) $\frac{dy}{dx} + 3y = e^x$

(C) $\left(\frac{dy}{dx}\right)^2 + 5y = 0$

(D) None of the above

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24. Let $I = \int_{-1}^{1} P_m(x)\, P_n(x)\, dx$, where $P_m(x)$ and $P_n(x)$ are Legendre polynomials, then

(A) $I = 0$, if $m \neq n$

(B) $I = 0$, if $m = n$

(C) $I = \frac{2}{2n+1}$, if $m \neq n$

(D) $I = \frac{2}{2n+1}$, if $m > n$

25. For n^{th} Legendre polynomial $C_n\left[\frac{d^n(x^2-1)^n}{dx^n}\right]$, the value of C_n is :

(A) $\frac{1}{n!\,2^n}$

(B) $\frac{n!}{2^n}$

(C) $n!\,2^n$

(D) None of the above

26. Which of the following is the Bessel's equation ?

(A) $z^2\left(\frac{d^2w}{dz^2}\right)+z\left(\frac{dw}{dz}\right)+(z^2+n^2)w=0$

(B) $z^2\left(\frac{d^2w}{dz^2}\right)-z\left(\frac{dw}{dz}\right)+(z^2-n^2)w=0$

(C) $z^2\left(\frac{d^2w}{dz^2}\right)+z\left(\frac{dw}{dz}\right)+(z^2-n^2)w=0$

(D) None of the above

27. What is the Wronskian of x and xe^2 ?

(A) 0

(B) $x - x^2e^x$

(C) x^2e^x

(D) None of the above

28. The partial differential equation obtained from $z = f(x) + e^y g(x)$ by eliminating the arbitrary functions is :

(A) $p = q$

(B) $t = q$

(C) $r = s$

(D) None of the above

29. What is the general solution of the partial differential equation $p - 2q = 3x^2 \sin(y + 2x)$?

(A) $x^3\sin(y + 2x) - z = \phi(y + 2x)$

(B) $\sin(y + 2x) = z$

(C) $\phi(y - 2x) = \cos(y + 2x)$

(D) None of the above

30. The partial differential equation

$$xy\frac{\partial^2 u}{\partial x^2}-(x^2-y^2)\left(\frac{\partial^2 u}{\partial x\partial y}\right)-xy\left(\frac{\partial^2 u}{\partial y^2}\right)+y\frac{\partial u}{\partial x}-x\frac{\partial u}{\partial y}=2(x^2-y^2)$$

(A) is parabolic at all points

(B) is hyperbolic at all points

(C) is elliptic at all points

(D) None of the above

31. Let $A = \{x \in R : x < 1\}$ and $B = \{x \in R : -1 < x \leq 3\}$. Then :

(A) Both A and B are bounded

(B) A is bounded above by 1 and its supremum $1 \in A$

(C) The supremum of A belongs to A and the infimum of B belongs to B

(D) The supremum of A does not belong to A and the supremum of B belongs to B

32. Which of the following is true ?

(A) The set of the real numbers is the neighbourhood of each of its points.

(B) The set Q of rational numbers is the neighbourhood of each of its points.

(C) The closed interval [a, b] is a neighbourhood of its end points a and b

(D) None of the above

33. What is the set of limit points for the set

$$S = \left\{-1, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{2}{3}, \frac{4}{3}, \ldots\right\}$$

(A) $\left\{-\frac{1}{2}, \frac{3}{2}\right\}$

(B) $\{-1, 1\}$

(C) $\{0, -1\}$

(D) None of the above

34. Which of the following is not true ?

(A) The set of all integers if countable.

(B) The set of all ordered pairs of integers is countable.

(C) The set of all real numbers is uncountable.

(D) The set of all rational numbers is uncountable.

35. What is the limit point of the sequence $\{a_n\}$, where $a_n = \frac{(-1)^{n-1}}{n!}$, $n \in N$.

(A) $-\frac{1}{2}$

(B) -1

(C) 0

(D) None of the above

36. If $a_n = n^2$ and $b_n = -n^2$, $n \in N$, then

(A) the sequence $(a_n + b_n)$ diverges

(B) the sequence $(a_n - b_n)$ converges

(C) the sequence $\left\{\frac{a_n}{b_n}\right\}$ diverges

(D) the sequence $\left\{\frac{a_n}{b_n}\right\}$ converges

37. $\lim_{n\to\infty}\left(\frac{2n-3}{n+1}\right)$ equals

(A) 0

(B) 1

(C) 2

(D) -3

38. Which of the following is not true ?

(A) If a sequence is convergent, it converges to a unique limit.

(B) Every convergent sequence is bounded.

(C) Every bounded sequence is convergent.

(D) Every bounded monotonic sequence is convergent.

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39. If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to 0} a_n$ equals :

(A) 0

(B) 1

(C) ∞

(D) None of the above

40. For any fixed value of n, the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ is :

(A) absolutely convergent

(B) divergent

(C) oscillates finitely

(D) oscillates infinitely

41. If $\sum a_n$ is a positive term series such that $\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = l$, then the series is convergent if :

(A) $l < 1$

(B) $1 < l < 3$

(C) $l \geq 3$

(D) None of the above

42. The series $A = \sum_{n=1}^{\infty} a_n$, where $a_n = \frac{(-1)^{n+1}}{\sqrt{n}}$. Then

(A) A is convergent

(B) A is divergent

(C) $\{a_n\}$ is monotonically increasing sequence

(D) the series $|a_n|$ is convergent

43. The radius of convergence and the exact interval of convergence of the power series $\sum \frac{(n+1)x^n}{(n+2)(n+3)}$ is :

(A) R = 0, [0, 1]

(B) R = 1, [–1, 1]

(C) R = ∞

(D) None of the above

44. Let $\{f_n\}$ be a sequence of functions such that $\lim_{n\to\infty} f_n = f(x)$, $x \in [a,b]$, and let $M_n = \sup_{x\in[a,\,b]} |f_n(x) - f(x)|$. Then $f_n \to f$ uniformly on [a, b] if and only if :

(A) $M_n \to 0$ as $n \to \infty$

(B) $M_n \to \infty$ as $n \to \infty$

(C) $M_n \to 1$ as $n \to \infty$

(D) None of the above

45. Let $\{f_n\}$ be a sequence, where $f_n(x) = \frac{nx}{1+n^2x^2}$, then

(A) $M_n \to 0$ as $n \to \infty$

(B) $M_n \to \frac{1}{2}$ as $n \to \infty$

(C) $M_n \to 1$ as $n \to \infty$

(D) None of the above

46. What is the identity element of the group of all integers Z with operation aob = a + b + 1, a, b $\in$ Z.

(A) 0

(B) 1

(C) –1

(D) None of the above

47. The set of all n × n non-singular matrices having their elements as rational numbers is :

(A) an infinite abelian group wrt matrix multiplication

(B) an abelian group wrt matrix addition

(C) an infinite non-abelian group wrt matrix multiplication

(D) None of the above

48. If $\alpha = (1\ 2\ 3\ 4\ 5)$ and $\beta = (2\ 3)\ (4\ 5)$ be two permutations of five symbols 1, 2, 3, 4, 5 then $\alpha\beta$ equals :

(A) (1 3 5)

(B) (2 4 5)

(C) (1 2 3)

(D) None of the above

49. Which of the following is the set of generators of the cyclic group $G = (\{0, 1, 2, 3, 4, 5\}, +6)$ are :

(A) {1, 2}

(B) {1, 5}

(C) {2, 5}

(D) None of the above

50. Which of the following is true ?

(A) The symmetric group P_3 of permutations of degree three is non-abelian, while its subgroup A_3 is abelian

(B) The symmetric group P_4 of permutations of degree four is abelian, while its subgroup A_4 is also abelian

(C) The symmetric group P_3 of permutations of degree three is non-abelian, while its subgroup A_3 is also non-abelian

(D) None of the above

51. Let $G = \{a\}$ be a cyclic group of order six and H be the subgroup generated by a^2. Then the order of the quotient group (G/H) is :

(A) 1

(B) 2

(C) 3

(D) 4

52. Which of the following is true for the group of order 45 ?

(A) G has a normal subgroup of order 9

(B) G has an element of order 9

(C) G is a non-abelian group

(D) G has no proper subgroup of order 5

53. What is the order of the element $\frac{2}{3}+z$ in the quotient group (G/Z) of the additive group of rationals ?

(A) 2

(B) 3

(C) 5

(D) 6

54. If the quotient group (G/Z), where Z is the centre of the group G, then

(A) G is abelian

(B) G is cyclic

(C) G is non-abelian

(D) None of the above

55. The set F = {0, 1, 2, 3, 4, 5, 6} under addition and multiplication modulo 7 forms :

(A) Commutative ring with zero divisors

(B) Non-commutative ring with unity

(C) Field

(D) None of the above

56. Let R[x] be the ring of polynomials of a ring R, then which of the following statements is/are true ?

(i) R is an integral domain if and only if R[x] is an integral domain.

(ii) If R is an integral domain, then deg [f(x) g(x)] = deg f(x) + deg g(x), where f(x), g(x) $\in$ R[x].

(A) (i) only

(B) (ii) only

(C) Neither (i) nor (ii)

(D) Both (i) and (ii)

57. What is the number of proper ideals in a field F ?

(A) At least one

(B) Zero

(C) Exactly one

(D) None of the above

58. $\lim_{n\to\infty}\left(a^{\frac{1}{n}}-1\right)n$ equals :

(A) log a

(B) 1

(C) ∞

(D) None of the above

59. What is the locus of the extremity of the polar subnormal of the curve r = f(θ) ?

(A) $\frac{dr}{d\theta}$

(B) $\theta-\frac{\pi}{2}$

(C) $r=f'\left(\theta-\frac{\pi}{2}\right)$

(D) None of the above

60. What is the angle of intersection of the curves r = a(1 + cos θ), r = b(1 − cos θ) ?

(A) $\frac{\pi}{2}$

(B) 0

(C) −1

(D) π